

Übungsaufgaben I, von 24.04.2001

Musterlösungen

1.

$$\alpha = \left(\frac{\partial V}{\partial T} \right)_p \approx \frac{\Delta V}{V \cdot \Delta T}$$
$$\Delta V = \alpha \cdot V \cdot \Delta T$$

$$\Delta V = 5.01 \cdot 10^{-5} \times 50 \cdot 10^{-6} \times 5 = 1.25 \cdot 10^{-3} \text{ m}^3$$

$$\Delta V = 125 \text{ mm}^3$$

2.

$$1 \text{ cal} \approx 4.184 \text{ J} \quad ; \quad 1 \text{ Watt} \leftrightarrow 1 \text{ J} \cdot \text{s}^{-1}$$

$$P_{av} = \frac{2500 \cdot 10^3 \cdot 4.184}{24 \cdot 60 \cdot 60 \text{ sec}}$$

$$P_{av} \approx 7.02 \text{ MW}$$

3.

$$\text{Anfangszustand:} \quad P_0 \cdot V_0 = n \cdot R \cdot T_0$$

$$\text{Endzustand:} \quad P_1 \cdot V_0 = n \cdot R \cdot T_1$$

$$\text{Wobei} \quad T_0 = 0^\circ\text{C} \quad 273,15 \text{ K}$$
$$\therefore T_1 = 296,15 \text{ K} \quad (23^\circ\text{C}),$$

$$P_0 \text{ (mm Hg)} = P_{\text{ext}} \text{ (mm Hg)} + h_0 \text{ (mm)},$$
$$P_1 \text{ (mm Hg)} = P_{\text{ext}} \text{ (mm Hg)} + h_1 \text{ (mm)};$$

$$P_{\text{ext}} : \text{extern Druck} = 1 \text{ atm}$$

$$V_0 = \frac{n \cdot R \cdot T_0}{P_0} = \frac{n \cdot R \cdot T_1}{P_1}$$

$$P_1 = \frac{P_0 \cdot T_1}{T_0}$$

$$h_1 = \frac{h_0 \cdot T_1}{T_0} + P_{\text{ext}} \cdot \frac{(T_1 - T_0)}{T_0}$$

$$h_1 \text{ (mm)} = 1,086 h_0 + 65 \text{ mm}$$

4.

$$f(x, y, z) = \frac{e^{-\lambda^2 x^2}}{\cos^2(3y - z^2)}$$

Vollständiges Differential $df(x, y, z) = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$

Benötigte Formeln: $\frac{d}{dx}(e^{f(x)}) = \frac{df(x)}{dx} e^{f(x)}$ $(e^f)' = f' e^f$
 $\left(\frac{1}{f}\right)' = -\frac{f'}{f^2}$;
 $(f^\alpha)' = \alpha f^{\alpha-1} f'$;
 $\cos(f)' = -f' \sin(f)$

Daraus folgt:

$$\frac{\partial f}{\partial x} = \frac{-2\lambda^2 x e^{-\lambda^2 x^2}}{\cos^2(3y - z^2)},$$
$$\frac{\partial f}{\partial y} = \frac{6e^{-\lambda^2 x^2}}{\cos^3(3y - z^2)} \sin(3y - z^2),$$
$$\frac{\partial f}{\partial z} = \frac{-4e^{-\lambda^2 x^2}}{\cos^3(3y - z^2)} z \sin(3y - z^2)$$

$$df(x, y, z) = \frac{-2e^{-\lambda^2 x^2}}{\cos^2(3y - z^2)} [\lambda^2 x dx - 3 \tan g(3y - z^2) dy + 2z \tan g(3y - z^2) dz]$$