Übungsaufgaben III, von 08.05.2001

Musterlösungen

1.

a) Ideales Gas: $pV = nRT$

$$\left.\begin{aligned}\left(\frac{\partial p}{\partial V}\right)_T &= \frac{-nRT}{V^2}\\ \left(\frac{\partial V}{\partial T}\right)_p &= \frac{nR}{p}\\ \left(\frac{\partial T}{\partial p}\right)_V &= \frac{V}{nR}\end{aligned}\right\} \quad \left(\frac{\partial p}{\partial V}\right)_T \cdot \left(\frac{\partial V}{\partial T}\right)_p \cdot \left(\frac{\partial T}{\partial p}\right)_V = \frac{-nRT}{V^2} \times \frac{nR}{p} \times \frac{V}{nR} = \frac{-nRT}{pV} = -1$$

b) Reales Gas: $(p + \frac{a}{V_m^2}) \cdot (V_m - b) = RT$

$$\left.\begin{aligned}\left(\frac{\partial p}{\partial V}\right)_T &= \frac{-RT}{(V-b)^2} + \frac{2a}{V^3}\\ \left(\frac{\partial V}{\partial T}\right)_p = \left(\frac{\partial T}{\partial V}\right)_p^{-1} &= \frac{1}{R} \times [\frac{(-2a)\times(V-b)}{V^3} + \left(p + \frac{a}{V^2}\right)]\\ \left(\frac{\partial T}{\partial p}\right)_V &= \frac{(V-b)}{R}\end{aligned}\right\}$$

$$\Rightarrow \left(\frac{\partial p}{\partial V}\right)_T \cdot \left(\frac{\partial V}{\partial T}\right)_p \cdot \left(\frac{\partial T}{\partial p}\right)_V = \left(\frac{-RT}{(V-b)^2} + \frac{2a}{V^3}\right) \times \frac{1}{\frac{1}{R} \times [\frac{(-2a)\times(V-b)}{V^3} + \left(p + \frac{a}{V^2}\right)]} \times \frac{(V - b)}{R} = -1$$

2.

Isotherm + ideales Gas => $\Delta U = 0$ in allen drei Fällen,
weil die Innere Energie eines idealen Gas nur von der Temperatur abhängt.

a) Isotherme reversible Volumenarbeit: $W = -nRT\ln\left(\frac{V_E}{V_A}\right)$

$$W = -1 \times 8{,}314 \times 273 \times \ln\left(\frac{44{,}8}{22{,}4}\right) = -1{,}57 \text{ kJ}$$

Nach dem ersten Hauptsatz: $Q = \Delta U - W = 0 + 1{,}57 = +1{,}57$ kJ

b) Expansion gegen einen konstanten Druck p_{ex}: $W = -p_{ex} \times \Delta V$

$$p_{ex} = p_E = \frac{nRT}{V_E} = \frac{1 \times 8{,}314 \times 273}{44{,}8 \times 10^{-3}} = 0{,}507 \times 10^5 \text{ Pa}$$

$$W = -0{,}507 \times 10^5 \times 22{,}4 \times 10^{-3} = -1{,}13 \times 10^3 \text{ J}$$

$$Q = \Delta U - W = 0 + 1{,}13 = +1{,}13 \text{ kJ}$$

c) $W = 0$ (freie Expansion);

$$Q = \Delta U - W = 0 - 0 = 0$$

3.

Adiabatische Expansion: $\Delta Q = 0$; $\Rightarrow dU = dW = -p \cdot dV$

$$pV^\kappa = \text{konst.} \qquad p = \frac{\text{konst.}}{V^\kappa}$$

$$\Rightarrow \Delta U = \int_I^{II} dU = -\int_{V_1}^{V_2} p \cdot dV = -\int_{V_1}^{V_2} \frac{\text{konst.}}{V^\kappa} \cdot dV$$

$$= \frac{\text{konst.}}{(\kappa - 1)}\left[\frac{1}{V_2^{(\kappa-1)}} - \frac{1}{V_1^{(\kappa-1)}}\right]; \quad \frac{\text{konst.}}{V^{\kappa-1}} = p \cdot V = n \cdot R \cdot T$$

$$\Delta U = \frac{n \cdot R}{(\kappa - 1)}[T_2 - T_1] = \frac{n \cdot R}{(\kappa - 1)} \cdot \Delta T$$

$$\kappa - 1 = \frac{C_P}{C_V} - 1 = \frac{C_P - C_V}{C_V} = \frac{n \cdot R}{C_V}$$

$$\Delta U = \frac{n \cdot R}{n \cdot R / C_V} \cdot \Delta T = C_V \cdot \Delta T$$

4.

$$\text{a) } W_A = -W_B;$$

$$W_B = -nRT\ln\left(\frac{V_E}{V_A}\right) \text{(isotherme reversible Volumenarbeit)}$$

$$W_B = -2\times 8{,}314\times 300\times \ln\left(\frac{1}{2}\right) = 3{,}46\times 10^3 \text{ J} \Rightarrow W_A = -3{,}46 \text{ kJ}$$

b) Isotherm + ideales Gas => $\Delta U_B = 0$

c) 1. HS: $Q_B = \Delta U_B - W_B = 0-(+3{,}46) = -3{,}46$ kJ

d) Endtemperatur im Teil A ≡ T_A^E

Im Teil A: Anfangszustand: $T_A^A = 300$ K , $P_A^A = ?$, $V_A^A = 2$ L

Endzustand: $T_A^E = ?$, $P_A^E = ?$, $V_A^E = 3$ L

Ideales Gasgesetz: $\frac{P_A^A \times V_A^A}{T_A^A} = \frac{P_A^E \times V_A^E}{T_A^E}; \frac{T_A^E}{T_A^A} = \frac{P_A^E \times V_A^E}{P_A^A \times V_A^A}$

3 unbekannte T_A^E, P_A^A, P_A^E aber eine Zusammenhang zwischen P_A^A und P_A^E:

Im Teil B: $V_B^E = \frac{1}{2} V_B^A$ $P_B^E = 2\times P_B^A$ und $P_B = P_A$ (Druckgleichheit in beiden Teilen)

Daraus folgt $P_A^E = 2\times P_A^A$ und $\frac{T_A^E}{T_A^A} = \frac{P_A^E}{P_A^A}\times\frac{V_A^E}{V_A^A} = 2\times\frac{3}{2} = 3$

Damit ist $T_A^E = 3\times T_A^A = 3\times 300 = 900$ K

$$\Delta U_A = n\times C_{V,m}\times \Delta T = 2\times 20\times 600 = +24 \text{ kJ}$$

e) 1 HS: $Q_A = \Delta U_A - w_A = 24-(-3{,}46) = 27{,}5$ kJ