Übungsaufgaben VII, von 10.12.2001

Musterlösungen

1.

$$N^2 \int \psi^* \psi dx = 1$$

$$N^2 \int_0^L \sin^2\left(\frac{\pi x}{L}\right) dx = 1 \qquad / \sin^2 b = \frac{1-\cos 2b}{2} /$$

$$\Rightarrow \quad N^2 \int_0^L \sin^2\left(\frac{\pi x}{L}\right) dx = \frac{1}{2} N^2 \int_0^L \left(1-\cos\frac{2\pi x}{L}\right) dx = \frac{1}{2} N^2 \left(x - \frac{L}{2\pi}\sin\frac{2\pi x}{L}\right)\Bigg|_0^L = \frac{1}{2} N^2 L = 1$$

für $\qquad N = \sqrt{\frac{2}{L}}$

2.

$\psi(x)$ Eigenfunktion von $\hat{C} \Rightarrow \hat{C}\psi(x) = c\psi(x)$
wobei $c = \text{Konstante} \neq f(x)$

a) $$\hat{A}\hat{B}\psi(x) = \left[\frac{d}{dx} x\right]\psi(x) = \frac{d}{dx}(x \cdot \psi(x)) = \psi(x) + x\frac{d\psi(x)}{dx} \qquad \text{(Produktregel)}$$

$$\hat{B}\hat{A}\psi(x) = \left[x\frac{d}{dx}\right]\psi(x) = x\frac{d\psi(x)}{dx}$$

$$\hat{C}\psi(x) = \left[\hat{A}\hat{B} - \hat{B}\hat{A}\right]\psi(x) = \hat{A}\hat{B}\psi(x) - \hat{B}\hat{A}\psi(x) = \psi(x) + x\frac{d\psi(x)}{dx} - x\frac{d\psi(x)}{dx} = \psi(x)$$

$$\Rightarrow \quad \hat{C}\,\psi(x) = 1 \cdot \psi(x)$$

$\psi(x)$ ist Eigenfunktion von C mit Eigenwert 1

b) $$\hat{A}\hat{B}\,\psi(x) = \left[\frac{d}{dx}x^2\right]\psi(x) = \frac{d}{dx}\left[x^2\psi(x)\right] = 2x\psi(x) + x^2\frac{d\psi(x)}{dx}$$

$$\hat{B}\hat{A}\,\psi(x) = \left[x^2\frac{d}{dx}\right]\psi(x) = x^2\frac{d\psi(x)}{dx}$$

$$\hat{C}\,\psi(x) = 2x\psi(x) + x^2\frac{d\psi(x)}{dx} - x^2\frac{d\psi(x)}{dx} = 2x\psi(x)$$

$$\Rightarrow \quad C\,\psi(x) = 2x\psi(x)$$

$\psi(x)$ ist keine Eigenfunktion von C, da $2x \neq$ Konstante

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3

a) $\langle x\rangle = \int \psi^* \hat{x} \psi dx$

$$\langle x\rangle = \frac{1}{L}\int_0^L e^{-i(kx-\omega t)} x e^{+i(kx-\omega t)} dx = \frac{1}{L}\int_0^L x dx = \frac{1}{L}\frac{1}{2}L^2$$

$$\Rightarrow \langle x\rangle = \frac{L}{2}$$

$$\hat{p} = -i\hbar\frac{d}{dx}$$

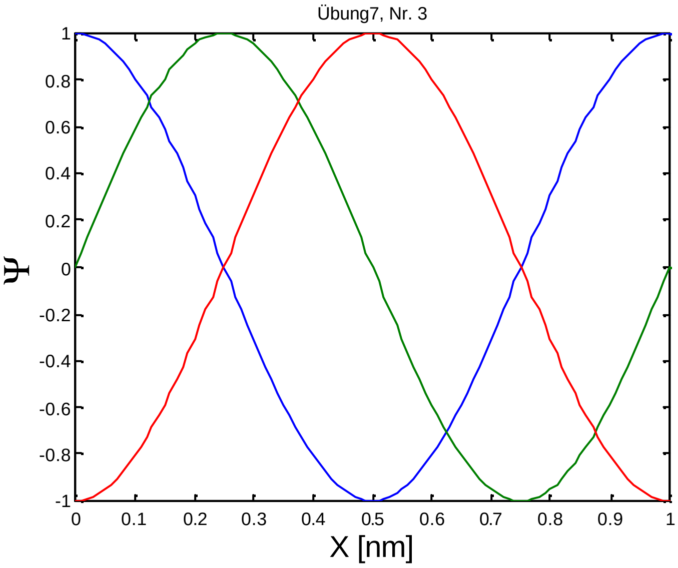
$$\langle p\rangle = \int \psi^* \hat{p}\psi dx = \int \psi^* - i\hbar\frac{d}{dx}\psi dx = -i\hbar\int \psi^*\frac{d\psi}{dx}dx$$

$$\psi(x) = \frac{1}{\sqrt{L}}e^{-i(kx-\omega t)} \qquad \frac{d\psi}{dx} = -ik\psi$$

$$\Rightarrow \langle p\rangle = -i\hbar\int \psi^*(-ik)\psi dx = -\hbar k$$

b)

für $\Psi(x,0)=\cos(2\pi x)$
für $\Psi(x,1)=\cos(2\pi x-0.5\pi)$
für $\Psi(x,2)=\cos(2\pi x-\pi)$



4.

$$\langle\hat{p}\rangle = \langle\psi_{end}|\hat{p}|\psi_{end}\rangle > 0.5 \qquad \psi_{end} = a\psi_1 + b\psi_2$$

$$\langle\hat{p}\rangle = \langle\psi_{end}|\hat{p}|\psi_{end}\rangle = \left\langle \psi_{end}\begin{pmatrix}00\\01\end{pmatrix}\times\begin{pmatrix}a\\b\end{pmatrix}\right\rangle = \left\langle (a\ b)\times\begin{pmatrix}0\\b\end{pmatrix}\right\rangle = b^2 > 0.5$$

$$\Rightarrow \langle\hat{p}\rangle = b^2 > 0.5$$

$$\Rightarrow b > \pm\sqrt{0.5}$$

$$\psi_{end} = n\hat{L}\psi(t_0)$$

$$n\hat{L}_{\alpha}\psi(t_0) = \hat{L}_{n.\alpha}\psi(t_0) = \begin{pmatrix} \cos(\alpha n) & \sin(\alpha n) \\ -\sin(\alpha n) & \cos(\alpha n) \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -\sin(\alpha n) \end{pmatrix}$$

$$-\sin(\alpha n) > \pm\sqrt{0.5}$$

$$\Rightarrow \quad \alpha n > \arcsin(\sqrt{0.5})$$

$$\Rightarrow \quad n > \frac{1}{\alpha}\arcsin(\sqrt{0.5})$$

$$n > 6$$