# Physikalische Chemie III für Lehramt

# Übungsblatt 2

(02.05.2023)

**Besprechung 04.05.2023**

## 1 Black body radiation

The intensity of the electromagnetic radiation emitted by an object due to its temperature depends on the wavelength $\lambda$ of the electromagnetic wave as follows:

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}. \tag{1}$$

In this expression $h$ is Planck's constant, $c$ is the speed of light, $k$ is Boltzmann's constant, and $T$ is the temperature of the body. The numerical values of the constants are

$$\begin{aligned} h &= 6.626\,070 \times 10^{-34}\ \mathrm{J\,s} \\ c &= 2.997\,925 \times 10^{8}\ \mathrm{m\,s^{-1}} \\ k &= 1.380\,649 \times 10^{-23}\ \mathrm{J\,K^{-1}}. \end{aligned} \tag{2}$$

**(a)** For $T = 2.725\,\mathrm{K}$, plot the radiation intensity for wavelengths in the interval from 0.1 mm to 10 mm.

According to your plot, at what wavelength is the intensity maximum?

The intensity curve that you analyzed corresponds exactly to the *cosmic microwave background radiation* that fills all space. Scientists study this radiation to understand the early universe.

**(b)** The surface of the Sun has a temperature of about 6000 K. At what wavelength does the Sun emit its peak intensity? (Hint: Check wavelengths in the interval from 20 nm to 2000 nm.)

## 2 Wave function

Let us assume that the wave function

$$\psi(x) = N e^{-\frac{x^2}{2}} \tag{3}$$

describes the quantum-mechanical state of a point particle in the interval $x \in (-\infty, \infty)$.

**(a)** Using the integral

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} \mathrm{d}x = \sqrt{\frac{\pi}{\alpha}} \tag{4}$$

calculate the value of the constant $N$ in eq. (3) such that the wave function is properly normalized. (This means that the *probability* to find the particle anywhere in the interval $x \in (-\infty, \infty)$ should equal one.)

**(b)** Plot the following functions:

$$\psi(x), \quad x\psi(x), \quad \psi'(x) = \frac{\mathrm{d}\psi(x)}{\mathrm{d}x}, \quad x\psi'(x), \quad \psi''(x). \tag{5}$$

**(c)** Is the wave function $\psi(x)$ in eq. (3) an *eigenfunction* of the position operator $\hat{x}$, of the momentum operator $\hat{p} = -\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}x}$, or of the kinetic energy operator $\hat{E}_K = \frac{\hat{p}^2}{2m}$ ($m$ is the mass of the particle)?

**(d)** Show that $\hat{x}\,\hat{p}\,\psi(x) \neq \hat{p}\,\hat{x}\,\psi(x)$. Use your calculations to determine the commutator

$$[\hat{x}, \hat{p}] = \hat{x}\,\hat{p} - \hat{p}\,\hat{x}. \tag{6}$$

**(e)** Calculate the following expectation values:

$$\langle\hat{x}\rangle, \quad \langle\hat{p}\rangle, \quad \langle\hat{x^2}\rangle, \quad \langle\hat{E}_K\rangle. \tag{7}$$