(1)

Berechnung der Larmor-Präzession von Spin

Benutze zeitabhängige SG: $\boxed{\hat{\mathcal{H}} \chi = i\hbar \frac{\partial \chi}{dt}}$

$\hat{\mathcal{H}}$: Zeemann Spin $\mathcal{H}$-Operator

χ : Spinwellenfktn.

Spinwellenfkt : $|\chi\rangle = c_1(t)|\alpha\rangle + c_2(t)|\beta\rangle$

$$= \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix}$$

Spin $\mathcal{H}$-Operator : $\hat{\mathcal{H}} = -\vec{\mu} \cdot \vec{B}_0$

mit $\vec{\mu} = \hbar \gamma_s \vec{\hat{S}}$

Spinoperator $\vec{\hat{S}} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$

mit $\hat{S}_x, \hat{S}_y, \hat{S}_z$ Pauli-Spin-Matrizen

$\hat{S}_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\hat{S}_x = \frac{1}{2}\begin{pmatrix} 0 & +1 \\ +1 & 0 \end{pmatrix}$ $\hat{S}_y = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$\vec{B}_0 = (0, 0, B_0)$

$\Leftrightarrow \hat{\mathcal{H}} = -\hbar \gamma_s B_0 \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(2)

$$i\hbar \begin{pmatrix} \dot{c}_1(t) \\ \dot{c}_2(t) \end{pmatrix} = -\frac{\hbar \gamma B_0}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix}$$

2 DG 1. Ordnung ~~(...)~~:

$$\dot{c}_1(t) = \frac{i\gamma B_0}{2} c_1(t) = i\frac{\omega_L}{2} c_1(t)$$

$$\dot{c}_2(t) = -\frac{i\gamma B_0}{2} c_2(t) = -i\frac{\omega_L}{2} c_2(t)$$

$$(\omega_L = \gamma B_0)$$

Löse ~~...~~ung:

$$c_1(t) = c_1(0) \cdot e^{\frac{i\omega_L}{2}t}$$

$$c_2(t) = c_2(0) \cdot e^{-\frac{i\omega_L}{2}t}$$

Wo steht Spin zu Zeit t ?

Berechne Erwartungswerte für S_x^*, S_y^*, S_z^*

$$\langle S_x \rangle = \langle \chi | \hat{S}_x | \chi \rangle$$

$$\langle S_y \rangle = \langle \chi | \hat{S}_y | \chi \rangle$$

$$\langle S_z \rangle = \langle \chi | \hat{S}_z | \chi \rangle$$

$$S_z = \frac{1}{2}\left(c_1^*(t), c_2^*(t)\right)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix}$$

$$= \frac{1}{2}\left(c_1^*(t), c_2^*(t)\right)\begin{pmatrix} c_1(t) \\ -c_2(t) \end{pmatrix}$$

$$= \frac{1}{2}\left(c_1^*(t) c_1(t) - c_2^*(t) c_2(t)\right) \quad \begin{cases} c_1^* = c_1^*(0) \cdot e^{-\frac{i\omega_L t}{2}} \\ c_2^* = c_2^*(0) e^{+\frac{i\omega_L t}{2}} \end{cases}$$

$$= \frac{1}{2}\left(c_1^*(0) c_1(0) - c_2^*(0) c_2(0)\right)$$

Zeitunabhängig, d. h. ändert sich nicht!

$$\langle S_x \rangle = \frac{1}{2}\left(c_1^*(t), c_2^*(t)\right)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix}$$

$$= \frac{1}{2}\left(c_1^*(t), c_2^*(t)\right)\begin{pmatrix} c_2(t) \\ c_1(t) \end{pmatrix}$$

$$= \frac{1}{2}\left(c_1^*(t) c_2(t) + c_2^*(t) c_1(t)\right)$$

$$= \frac{1}{2}\left(c_1^*(0) c_2(0) \cdot e^{-i\omega_L t} + c_2^*(0) c_1(0) \cdot e^{i\omega_L t}\right)$$

$$\langle S_y \rangle = \frac{1}{2} \left(c_1^*(t), c_2^*(t) \right) \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix}$$

$$= \frac{i}{2} \left(c_1^*(t), c_2^*(t) \right) \begin{pmatrix} -c_2(t) \\ c_1(t) \end{pmatrix}$$

$$= \frac{i}{2} \left(-c_1^*(0)\, c_2(0)\, e^{-i\omega_L t} + c_2^*(0)\, c_1(0) \cdot e^{+i\omega_L t} \right)$$

<u>Wähle nun Anfangsbedingungen:</u>

$\boxed{A}$ $\quad \chi(0) = |\alpha\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$S_z = \frac{1}{2}(1) \qquad (+1/2) \text{ Spin up}$$

$$S_x = \frac{1}{2}\left(1 \cdot 0\, e^{-i\omega_L t} + 0 \cdot 1 \cdot e^{i\omega_L t} \right) = 0$$

$$S_y = \frac{i}{2}\left(-1 \cdot 0 \cdot e^{-i\omega_L t} + 0 \cdot 1 \cdot e^{i\omega_L t} \right) = 0$$

stationärer Zustand, keine x, y - Komponente

nur z - <u>EW</u>

$\boxed{B}$ Wähle $\chi(0) = \frac{1}{\sqrt{2}}(1, 1)$ Überlagerungszustand (nach $\pi/2$ Puls)

keine z-Komponente

$$S_z = \frac{1}{2}\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\right) = \underline{0}$$

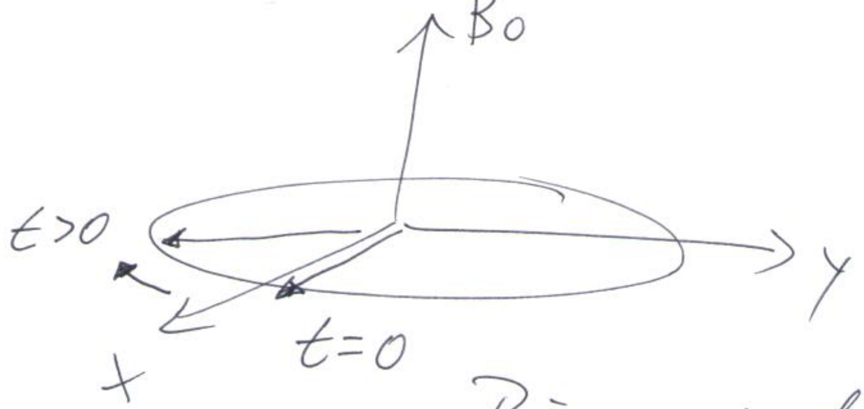
$$S_x = \frac{1}{2}\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} e^{-i\omega_L t} + \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\cdot e^{i\omega_L t}\right)$$

$$= \frac{1}{4}\cdot 2\cdot\cos(\omega_L t) \quad \text{(Euler-Satz)}$$

$$= \frac{1}{2}\cos(\omega_L t)$$

$$S_y = \frac{i}{2}\left(-\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} e^{-i\omega_L t} + \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} e^{i\omega_L t}\right)$$

$$= -\frac{1}{2}\sin(\omega_L t)$$



Präzession in der x-y Ebene ($\perp B_0$) mit Larmorfrequenz ω_L

mit MW – Einstrahlung

$$\hat{\mathcal{H}}(t) = -\hat{\vec{\mu}} \cdot \vec{B}(t)$$

$$= -\hbar\gamma \hat{\vec{S}} \left(\vec{B}_0 + \vec{B}_1(t) \right)$$

$$= +\hbar\omega_L \hat{S}_z + \hbar\omega_1 \left[\cos\omega_{MW} t \, \hat{S}_x + \sin\omega_{MW} t \, \hat{S}_y \right]$$

$$\hat{\mathcal{H}}(t) = \frac{\hbar}{2} \begin{pmatrix} \omega_L & \omega_1 e^{-i\omega_{MW} t} \\ \omega_1 e^{i\omega_{MW} t} & -\omega_L \end{pmatrix}$$

gekoppelte DG

$$\left\{ \begin{aligned} i \dot{c}_1(t) &= \frac{\omega_L}{2} c_1(t) + \frac{\omega_1}{2} e^{-i\omega_{MW} t} c_2(t) \\ i \dot{c}_2(t) &= \frac{\omega_1}{2} e^{i\omega_{MW} t} c_1(t) - \frac{\omega_L}{2} c_2(t) \end{aligned} \right.$$

Definiere neue Fkt.:

{entspricht rot. KS}

$$b_1(t) = e^{i\omega_{MW} t/2} c_1(t)$$

$$b_2(t) = e^{-i\omega_{MW} t/2} c_2(t)$$

$$i \dot{b}_1(t) = -\frac{\Delta\omega}{2} b_1(t) + \frac{\omega_1}{2} b_2(t)$$

$$i \dot{b}_2(t) = \frac{\omega_1}{2} b_1(t) + \frac{\Delta\omega}{2} b_2(t)$$

Dies entspricht einer Zeitabhäng. SG mit

$$\chi_r(t) = e^{i\omega_{MW} t \hat{S}_z} \cdot \chi_L(t)$$

und $$\hat{\mathcal{H}}_r = \frac{\hbar}{2}\begin{pmatrix} -\Delta\omega & \omega_1 \\ \omega_1 & \Delta\omega \end{pmatrix} = \underbrace{\Delta\omega S_z}_{\text{Sbchierter Zeemann effekt}} + \underbrace{\omega_1 S_x}_{\text{Zeitunab. } B_1}$$

$(\Delta\omega = \gamma B_0 - \omega_{MW})$

on - Resonanz $\Delta\omega = 0 \; (\omega_{MW} = \omega_L)$

$$\left.\begin{aligned} \dot{b}_1(t) &= -i\frac{\omega_1}{2} b_2(t) \\ \dot{b}_2(t) &= -i\frac{\omega_1}{2} b_1(t) \end{aligned}\right\} \text{gekoppelte LDG 1. Ordnung}$$

{Löse durch diff + Einsetzen}

$$\ddot{b}_1(t) = -i\frac{\omega_1}{2}\dot{b}_2(t) = -\frac{\omega_1^2}{4} b_1(t)$$

Lösungsansatz : $b_1(t) = a \cdot e^{i\lambda t}$

$$\dot{b}_1(t) = i a \lambda e^{i\lambda t}$$

$$\ddot{b}_1(t) = -a\lambda^2 e^{i\lambda t}$$

$$-a\lambda^2 e^{i\lambda t} = -\frac{\omega_1^2}{4} a \cdot e^{i\lambda t}$$

$$\Rightarrow \lambda^2 = \frac{\omega_1^2}{4} \qquad \boxed{\lambda = \pm \frac{\omega_1}{2}}$$

General solution for $b_1(t)$ and $b_2(t)$:

$$b_1(t) = a_1 \cdot e^{+i\frac{\omega_1}{2}t} + a_2 e^{-i\frac{\omega_1}{2}t}$$

$$b_2(t) = d_1 \cdot e^{+i\frac{\omega_1}{2}t} + d_2 e^{-i\frac{\omega_1}{2}t}$$

Anfangsbedingungen: a) $\chi(0) = \alpha\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$c_1 = 1,\ c_2 = 0 \rightsquigarrow b_1(0) = 1,\ b_2(0) = 0$

$\hookrightarrow \boxed{a_1 + a_2 = 1,\ d_1 + d_2 = 0}$

b) [illegible]

+ Normierung!

$\downarrow$

$\cancel{a_1 + a_2} = a_1 = a_2 = \frac{1}{2},\ d_1 = \frac{1}{2},\ d_2 = -\frac{1}{2}$

$$b_1(t) = \frac{1}{2}\left(e^{i\frac{\omega_1}{2}t} + e^{-i\frac{\omega_1}{2}t}\right) \left\{= \cos(\omega_{1/2}\, t)\right\}$$

$$b_2(t) = \frac{1}{2}\left(e^{i\frac{\omega_1}{2}t} - e^{-i\frac{\omega_1}{2}t}\right) \left\{= i\sin(\omega_{1/2}\, t)\right\}$$

⑨

$$\langle S_z \rangle = \frac{1}{2}\left(\cos\frac{\omega_1 t}{2}, -i\sin\frac{\omega_1 t}{2}\right)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \cos\frac{\omega_1 t}{2} \\ i\sin\frac{\omega_1 t}{2} \end{pmatrix}$$

$$= \frac{1}{2}\left(\cos\frac{\omega_1 t}{2}, -i\sin\frac{\omega_1 t}{2}\right)\begin{pmatrix} \cos\frac{\omega_1 t}{2} \\ -i\sin\frac{\omega_1 t}{2} \end{pmatrix}$$

$$= \frac{1}{2}\underbrace{\left(\cos\frac{\omega_1 t}{2}^2 - \sin\frac{\omega_1 t}{2}^2\right)}_{\cos(\omega_1 t)} = \frac{1}{2}\cos(\omega_1 t)$$

$$\langle S_y \rangle = \frac{i}{2}\left(\cos\frac{\omega_1 t}{2}, -i\sin\frac{\omega_1 t}{2}\right)\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \cos\frac{\omega_1 t}{2} \\ i\sin\frac{\omega_1 t}{2} \end{pmatrix}$$

$$= \frac{i}{2}\left(\cos\frac{\omega_1 t}{2}, -i\sin\frac{\omega_1 t}{2}\right)\begin{pmatrix} -i\sin\frac{\omega_1 t}{2} \\ \cos\frac{\omega_1 t}{2} \end{pmatrix}$$

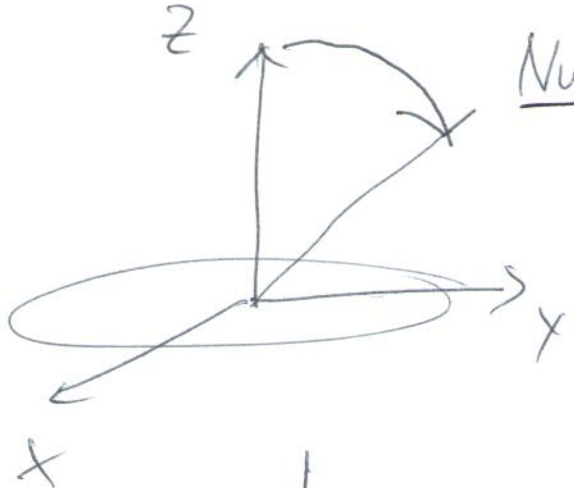
$$= -\frac{i^2}{2}\left(\cos\frac{\omega_1 t}{2}\sin\frac{\omega_1 t}{2} + \sin\frac{\omega_1 t}{2}\cos\frac{\omega_1 t}{2}\right)$$

$$= \frac{1}{2}\left(\underbrace{2\cos\frac{\omega_1 t}{2}\sin\frac{\omega_1 t}{2}}_{\sin(\omega_1 t)}\right) = \frac{1}{2}\sin(\omega_1 t)$$

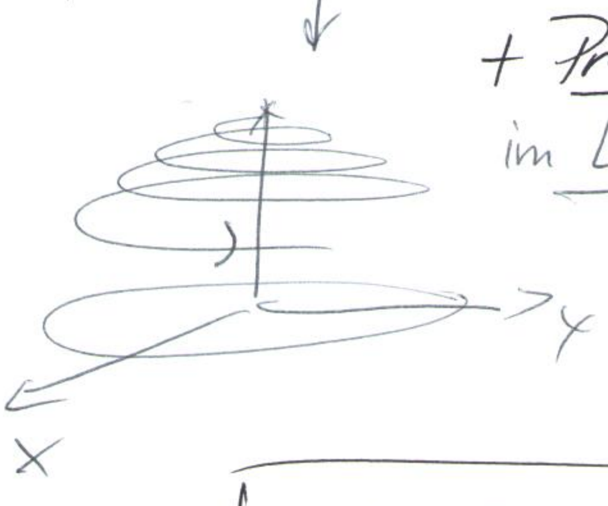
$$\langle S_x \rangle = \frac{1}{2}\left(\cos\frac{\omega_1 t}{2}, -i\sin\frac{\omega_1 t}{2}\right)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\cos\frac{\omega_1 t}{2}\\ i\sin\frac{\omega_1 t}{2}\end{pmatrix}$$

$$= \frac{1}{2}\left(\cos\frac{\omega_1 t}{2}, -i\sin\frac{\omega_1 t}{2}\right)\begin{pmatrix}i\sin\frac{\omega_1 t}{2}\\ \cos\frac{\omega_1 t}{2}\end{pmatrix}$$

$$= \frac{i}{2}\left(\cos\frac{\omega_1 t}{2}\sin\frac{\omega_1 t}{2} - \sin\frac{\omega_1 t}{2}\cos\frac{\omega_1 t}{2}\right) = 0$$



Nutation mit ω_1 im rot. KS
Resi-Oszillation



+ Präzession mit ω_L
im Laborsystem

siehe: $\boxed{\text{Cohen Tannoudji Quantum Mechanics}}$